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MULTI-OBJECTIVE MATHEMATICAL OPTIMIZATION IN THE SMART CITY SUPPLY PLANNING PROBLEM

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Abstract

The mathematical model of multi-objective decision-making is of key importance for the development of the smart city model. Today's accelerated development of information technologies is closely related to the development of mathematical models. This paper deals with the problem of smart city supply planning and a mathematical model that could describe that supply.

Keywords: *mathematical optimization, smart city, mathematical model, information technology, supply*

Introduction

Multi-objective optimization (MOO) is encountered in everyday life in fields such as mathematics, engineering, automotive and many others. Different types of MOO problems go along with alignment methods, global criterion method, weighted sum method, ε-constraint method, etc. Some of the IOC calculation methods show that complex solving methods and difficult mathematical equations are used.

In response to the ongoing urbanization trend seen around the world, a phenomenon known as smart cities has emerged. In smart cities, new technologies are carefully woven into the fabric of urban life to modify many aspects of daily life. This is done to make the city more sustainable. [1] One of the most significant obstacles that cities have to face is the optimization of all segments of life. This need is gaining importance as the demand for optimization solutions that are not only efficient but also comprehensively applicable continues to grow. As the number of people living in urban areas increases, there is an increasing need for living conditions that are carefully planned, adaptable and intelligent.[2] What gives this imperative its importance is the multiple impact it has on the economic vitality, health, environment and overall well-being of the urban population. This is what makes mathematical optimization so applicable in this segment of urbanization. In the never-ending quest for economic vitality, smart cities recognize the very important role played by an optimized lifestyle and work.[3] Smart cities aim to minimize the amount of wasted time

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and stress associated with daily travel by streamlining processes among its residents. In turn, this leads to an urban economy that is not only more vibrant, but also more dynamic.[4] Mathematical modeling, as a methodology used in various scientific fields, has a great impact on the sustainable development of smart cities, improving living and working conditions. Because of all its benefits, it continuously motivates scientists to develop and devise new models that correspond to more realistic situations and the current state of society. In this paper, attention is focused on the presentation of the mathematical optimization model and its possible application in the concept of smart cities.

Applied solutions

Since the birth of the idea of smart cities, we have had a great race of science and technology. Numerous questions arise, especially which already existing solutions can be implemented and which can be improved. There is a constant development of intelligent solutions that imperceptibly creep into every pore of life. From the first smart house to the first smart city, about a quarter of a century has passed. Today, we aspire to introduce smart solutions into the existing infrastructure and improve all forms of life and business. Examples of mathematical optimization are numerous and cannot be presented in one article. Here we will highlight a few ubiquitous applications.

In smart cities, new technologies are carefully woven into the fabric of urban life to modify many aspects of daily life. This is done to make the city more sustainable. One of the most significant obstacles that city centers have to face is the optimization of the public transport network. The demand for transport solutions that are not only efficient but also good for the environment continues to grow. As the number of people living in urban areas continues to grow, there is an increasing need for public transport that is carefully planned, adaptive and intelligent.[2] In the never-ending quest for economic vitality, smart cities recognize the very important role played by an optimized public transport infrastructure.[3] By promoting the use of environmentally friendly vehicles, investing in alternative energy sources and strategic planning of transit routes, the goal of smart cities is to reduce the negative impact that urban mobility has on the environment. The successful completion of this goal is achieved through the implementation of smart city projects.[5] As a direct result of such improvements, people living in the neighborhood will find that the air they breathe is cleaner and their living conditions will improve.[1]

Basic concepts and principles of multi-criteria optimization methods and approaches to multiple criteria decision-making are the basis of all solutions in the development of smart cities. The algorithms used are classified based on the role of the decision maker, and four different classifications are made. Algorithms such as the weighting method, the epsilon constraint method, the scalarization function method of achievement, and the normal cutoff method. Basic concepts and principles of multi-criteria optimization methods and approaches to multiple criteria decision-making are the basis of all solutions in the development of smart cities. The algorithms used are classified based on the role of the decision maker, and four different classifications are made. Algorithms such as the weighting method, the epsilon constraint method, the scalarization function method of achievement, and the normal cutoff method. The current trends of urbanization and growing economy bring with them increasing levels of traffic congestion and city authorities have to come up with new strategies to solve such problems. Multi-level distribution is already a well-known strategy used by companies.

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An example of the application of optimization solutions in traffic is two-echelon vehicle routing (2E-VRP) and reflects the perspective of one provider, regardless of the routing decisions of other parties. The lack of coordination between providers executing their individual schedules and, consequently, the lack of a holistic approach to urban transport can cause further problems. In addition, different stakeholders (government, businesses, residents) may have conflicting goals. In the paper, Eitzen presents the first multi-objective formulation of a heterogeneous multi-provider (or multi-commodity) vehicle 2E-VRP, from the perspective of city administration in the context of the urban goods movement, demonstrating with didactic examples the potential benefits of this approach to all involved parties, while taking into account potentially conflicting goals.[6]
Citizens are becoming more aware of the Internet of Things (IoT), which is a growing network of digital sensors, smart gadgets, and smart home appliances, and prompt attention to any of these technologies may enrich people's quality of life. For instance, in the near future, long-lasting batteries, which are now being developed, will be capable of a lot of work and will be able to charge themselves using daylight, temperature, or motion [7]. This is a perfect illustration of the solutions, which are now being developed for smart cities. Furthermore, each city's smart city goals and major implementations are unique, and these smart gadgets and systems will continue to be employed in the future. [8]

Given the growing population in metropolitan areas, it is difficult to provide adequate services and environmental requirements. As a result, mathematical optimization models could not be applied without IoT technology. These technologies are considered a promising tool for establishing a functional smart city [9, 10]. Despite the fact that designing an IoT architecture is a difficult endeavor, this information management technology is really widely used in smart cities. In this platform, a wide range of gadgets, network layer solutions and applications, connected with mathematical models, must be used.[11] Furthermore, those solutions should be adaptable to different intelligent locations[12]: floating cities, which are very common and range from cities and islands floating on water to those that would float in the planet's atmosphere, and smart cities are emerging as a kind of smart environment on the planet.

Pareto model

The problem of decision-making in the IOC enables a compromise (tradeoff) on some contradictory issues. The IOC was introduced by Vilfredo Pareto. There is an objective function vector in MOO. Each objective function vector is a function of the solution vector. In IOC, there is not one best solution for all purposes, but several solutions.

Mathematically, the equations of the MOO problem can be written as follows [13]:

$$\min/\max\ f_1(x), f_2(x), \ldots, f_n(x) \qquad (1)$$

subject to: $x \in U$,

where x is solution, n is the number of objective functions, U is feasible set, $fn(x)$ is n th objective function, and min/max is combined object operations.

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In the MOO, there is a multi-dimensional space of the objective function vector and the decision variable space of the solution vector. In every x solution in the decision variable space there is a point on the objective function space. The mapping between the solution vector and the objective function vector can be seen in Figure 1.[14]

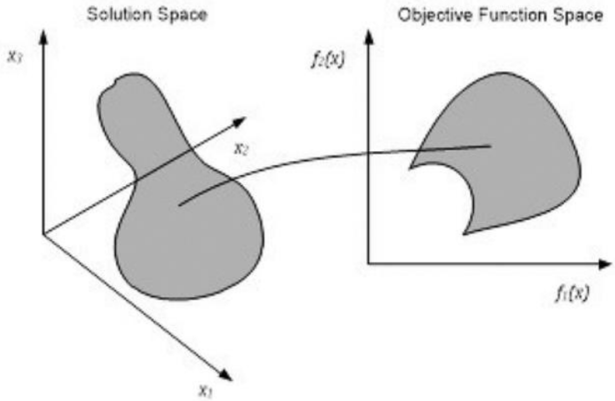

Figure 1. Mapping solution space and objective function space.

Given this mapping, the convexity of a solution space and an objective function space is crucial in determining the problem-solving algorithm. If the MOO problem is convex, then there are many algorithms that can be used to solve the problem well. MOO problems are said to be convex if all the objective functions and solution area are also convex. Meanwhile, the objective function is said to be convex if it satisfies the following equation: [15]

$$f(\theta x + (1-\theta)y) \leq \theta f(x) + (1-\theta) f(y), \quad (2)$$

with x, y domain f and value which is 01. To understand Equation (2) better, it is inferred that the line between (x,f(x)) and (y,f(y)) that runs between x to y is above the f graph. This can be seen in Figure 2.

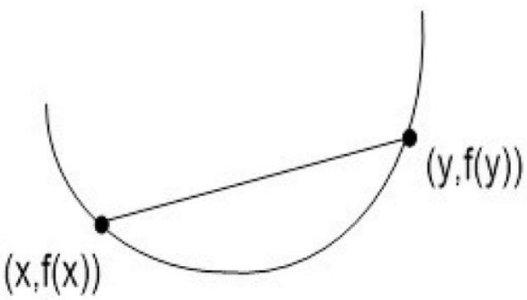

Figure 2. Convex function.

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The solution of the MOO problem can be classified into two namely the Pareto method and scalarization. [16] The Pareto method and scalarization are different. The Pareto method is used if the desired solutions and performance indicators are separate and produce a compromise solution (tradeoff) and can be displayed in the form of Pareto optimal front (POF). Meanwhile, the scalarization method is a performance indicators component that forms a scalar function which is incorporated in the fitness function. [17]
Mathematically, the MOO problem using the Pareto method can be written as follows [13]:

$$f_1,opt=min\ f_1(x)f2,opt=min\ f2(x)..f_n,opt=max\ f_n(x). \qquad (3)$$

The Pareto method keeps the elements of the solution vectors separate (independent) during optimization and the concept of dominance is there to differentiate the dominated and non-dominated solutions. The dominance solution and optimal value in MOO are usually achieved when one objective function cannot increase without reducing the other objective function.
The scalarization method makes the multi-objective function create a single solution and the weight is determined before the optimization process. The scalarization method incorporates multi-objective functions into scalar fitness function as in the following equation [18]:

$$F(x)=w_1f_1(x)+w_2f_2(x)+\ldots+w_nf_n(x). \qquad (5)$$

The weight of an objective function will determine the solution of the fitness function and show the performance priority.
The selection of relays on ad hoc networks with MOO uses the Pareto method.[13] The problems to be optimized are the criteria used are throughput, load balancing, power consumption. The ad hoc network model that is used is a model outside the building and inside the building.
The next application of the scalarization method is the creation of a simple and easy-to-understand cooperative protocol with multi-objective criterion that takes into account the *source–destination (S–D) conditions with the amplify and forward method.[13] In addition, this cooperative diversity protocol guarantees full diversity based on three objective criteria. These criteria are signal-to-noise ratio, power consumption, and load variance.

Conclusion

Problems in IOC are numerous and can be found in various areas of human life. Their application to the concept of smart cities is incalculable. New applications are found every day, but the question is whether with the development of computing, everything will come to the fore or if it will be overcome after just a few hours. There are many methods to solve the IOC problem. If we consider the generality of the application of the IOC, there are two conclusions. First, there are two MOO methods that do not require complex mathematical equations so that the problem becomes simplified, namely Pareto and scalarization methods. In the Pareto method, there are dominant solutions and non-dominated solutions that can be described in POF. A non-dominated solution is obtained through a continuously updated algorithm. The scalarization method creates multiple objective functions into a single

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solution, and the weights are determined first. The weights of the scalarization method are equal weights, ROC weights, and RS weights. Second, the solution from the Pareto method is a performance indicator that forms the MOO separately and produces a compromise solution and can be displayed in the form of a POF.
Further research will lead to new possibilities of applying these and other IOC methods to all concepts of life, in order to improve living conditions, preserve the environment and better use all resources.
MOO or multi-objective optimization refers to finding optimal solution values for more than one desired objective. The motivation for using MOO is that it does not require complicated equations, thereby simplifying the problem. [19] Due to its convenience, this optimization model can be approached not only by engineers, but also by economists and lawyers who will recognize the importance of investing in this segment of mathematics in their decisions.

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