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DESIGN OF ALLPASS-BASED IIR FULLBAND DIFFERENTIATORS USING LINEAR PROGRAMMING

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Abstract

5G technology is one of the main prerequisites for the Smart Cities paradigm. 5G transceivers can notably benefit from the utilization of sophisticated signal processing techniques such as advanced digital differentiators due to the requirements for high data rates, low latency, and efficient spectrum utilization. This paper investigates the design of all-pass based infinite impulse response full band differentiators using linear programming. Obtained differentiators.
are optimal in the sense that relative error of the magnitude response is minimized in the Chebyshev sense. Since the starting optimization problem is non-linear, it is divided into several sub-problems that can be easily solved. As compared to existing all-pass based solutions, proposed differentiators have an additional design parameter that allows a further decrease in the magnitude response error.

Keywords: *allpass filter, fullband differentiators, linear programming, parallel allpass structure, magnitude response error minimization*

Introduction

In the design of 5G transceivers, the choice of implementing ALLPASS-based IIR differentiators would typically be made considering factors like specific performance requirements, power consumption, implementation complexity, and cost. Digital differentiators are required in various applications where time derivative of input signal needs to be computed. These applications include physiological signal processing, edge detection in images, analysis of radar and sonar signals, and compensation in control systems. As other

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types of filters, digital differentiators can be also designed as infinite impulse response (IIR) or finite impulse response filters. While IIR fullband differentiators cannot have linear phases, they are preferred in many practical applications due to considerably lower filter order and group delay values.

IIR fullband differentiators can be designed by inverting the transfer function of the IIR integrator, followed by stabilization of unstable poles [1, 2, 3]. Starting from the existing transfer function, methods of the second approach tend to optimize some existing differentiator transfer function [4, 5, 6] using classical and metaheuristic optimization methods. Method presented in [7] formulates the design problem as convex constrained optimization problem in unknown zeros and poles locations such that its solution minimizes the group delay deviation under the constraint that maximum relative magnitude response error is below some prescribed value. Differentiators designed by minimization of the L_2 norm of magnitude response error are discussed in [8, 9]. Finally, starting point of the allpass-based design methods [10, 11, 12, 13] is assumption that digital differentiators can be realized by parallel connection of two allpass filters.

In this paper, a new approach for the design of IIR fullband digital differentiators using parallel allpass structure, where one of the parallel allpass branches is pure delay, is presented. While orders of allpass filter in parallel allpass branches differ by one, the case which is already discussed in [10, 11, 12], the novelty of this study is the introduction of additional design parameter that allows further decrease of the magnitude response error. Further, design problem is formulated in a new way and the optimal solution is obtained by solving several subproblems.

Problem formulation

Transfer function of considered allpass-based IIR fullband differentiators is assumed to be of the following form

$$H(z) = \frac{\gamma}{2}\left[z^{-(N-1)} - A_N(z)\right], \quad \gamma > 0, \tag{1}$$

where $A_N(z)$ is transfer function of N-th order stable allpass filter

$$A_N(z) = z^{-N}\frac{P(z^{-1})}{P(z)}, \quad P(z) = 1 + \sum_{k=1}^{N} a_k z^{-k}, \tag{2}$$

while the value of γ equals the maximum magnitude response at $\omega = \pi$ as will be shown later. Notably, transfer function of fullband differentiators considered in [10] can be also expressed in the form given by (1), however these differentiators have one degree of freedom less as γ is fixed and equal to π.

Substituting $z = e^{j\omega}$ in (1), magnitude and phase responses of considered allpass-based IIR fullband differentiators can be expressed as

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$$|H(e^{j\omega})| = \gamma \sin\frac{-(N-1)\omega-\phi(\omega)}{2}, \tag{3}$$

$$\theta(\omega) = \arg\{H(e^{j\omega})\} = \frac{\pi}{2} + \frac{\phi(\omega)-(N-1)\omega}{2}, \tag{4}$$

where $\phi(\omega)$ is the phase response of allpass filter $A_N(z)$,

$$\phi(\omega) =- N\omega - 2\arg\{P(e^{j\omega})\}. \tag{5}$$

As the phase response of stable allpass filter is monotonically decreasing function of frequency for $\omega \in [0, \pi]$, while $\phi(0) = 0$ and $\phi(\pi) =- N\pi$ [14], it follows that $|H(e^{j0})| = 0$, $\theta(0) = \pi/2$, and $H(e^{j\pi}) = \gamma \exp\{-j(N-1)\pi\}$. Therefore, the maximum of the magnitude response equals γ and it occurs at $\omega = \pi$, while the average group delay of proposed differentiators equals

$$\tau = \frac{\theta(0)-\theta(\pi)}{\pi} = N - \frac{1}{2}, \tag{6}$$

regardless of the allpass filter coefficients.

Since IIR fullband differentiators are usually designed such that relative error of the magnitude response is minimized [7], in this paper, an algorithm for determination of the unknown coefficients vector

$$\boldsymbol{a} = [a_1\, a_2 \,...\, a_N]^T \tag{7}$$

and value of the parameter γ, is derived such that relative error of the magnitude response

$$\varepsilon(\omega) = \omega^{-1}(|H(e^{j\omega})| - \omega) \tag{8}$$

is minimized in the Chebyshev sense. Note that (3), and consequently (8), is valid only if coefficients of the allpass filter are such that right-hand side of this equation is positive for $\omega \in (0,\ \pi)$. Hence, the first set of constraints that needs to be imposed on vector $\boldsymbol{a}$ are

$$\sin\frac{-(N-1)\omega-\phi(\omega)}{2} > 0, \tag{9}$$

for $\omega \in (0,\ \pi)$.

DESIGN ALGORITHM

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In this section, a problem of determination of $\boldsymbol{a}$ and γ, such that $\varepsilon(\omega)$ is minimized in the Chebyshev sense, is solved by solving several subproblems. First a method to obtain the allpass filter coefficients $\boldsymbol{a}$ such that $\varepsilon(\omega)$ satisfies

$$-\delta \leq \varepsilon(\omega) \leq \delta, \tag{10}$$

for $\omega \in [0, \pi]$ and assuming δ and γ known is discussed. As will be shown, this subproblem can be formulated as the linear programming one. Then, assuming γ known, determination of the lowest δ, i.e. the Chebyshev norm $\delta_C = \delta(\gamma)$, is considered. Finally, γ is determined such that obtained δ_C is minimal for the given filter order.

Assuming γ and δ known, allpass-based IIR fullband differentiator design problem reduces to determination of the allpass filter coefficients such that (10) is satisfied, i.e. such that

$$\frac{\omega(1-\delta)}{\gamma} \leq \sin\frac{-(N-1)\omega-\phi(\omega)}{2} \leq \frac{\omega(1+\delta)}{\gamma}, \tag{11}$$

under the constraints given by (9). Obviously, δ should satisfy $\delta \geq 1-\gamma/\pi$ as $\varepsilon(\pi) = \gamma/\pi - 1$, regardless the coefficients values. Imposing additional constraint that magnitude response has only one extremal value occurring at $\omega = \pi$, i.e. that

$$\cos\frac{-(N-1)\omega-\phi(\omega)}{2} > 0, \tag{12}$$

for $\omega \in (0, \pi)$, previous equations can be formulated as linear inequality constraints in unknown coefficients vector $\boldsymbol{a}$. Namely, as

$$\sin\frac{-(N-1)\omega-\phi(\omega)}{2} = \sin\left(\arg\left\{e^{j\omega/2}P(e^{j\omega})\right\}\right) = \frac{R(\omega)}{\sqrt{R^2(\omega)+Q^2(\omega)}} \tag{13}$$

$$\cos\frac{-(N-1)\omega-\phi(\omega)}{2} = \cos\left(\arg\left\{e^{j\omega/2}P(e^{j\omega})\right\}\right) = \frac{Q(\omega)}{\sqrt{R^2(\omega)+Q^2(\omega)}} \tag{14}$$

Where

$$R(\omega) = \sin\frac{\omega}{2} + \sum_{k=1}^{N} a_k \sin\left(\frac{\omega}{2} - k\omega\right) = \sin\frac{\omega}{2} + \boldsymbol{s}(\omega)\boldsymbol{a}, \tag{15}$$

$$Q(\omega) = \cos\frac{\omega}{2} + \sum_{k=1}^{N} a_k \cos\left(\frac{\omega}{2} - k\omega\right) = \cos\frac{\omega}{2} + \boldsymbol{c}(\omega)\boldsymbol{a}, \tag{16}$$

inequalities given by (9) and (12) can be expressed as

$$\boldsymbol{s}(\omega)\boldsymbol{a} > -\sin\frac{\omega}{2}, \tag{17}$$

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and

$$\boldsymbol{c}(\omega)\boldsymbol{a} > -\cos\frac{\omega}{2}, \tag{18}$$

respectively, while (11) can be formulated as

$$\left[\boldsymbol{s}(\omega)\sqrt{1-\frac{\omega^2(1+\delta)^2}{\gamma^2}}-\boldsymbol{c}(\omega)\frac{\omega(1+\delta)}{\gamma}\right]\boldsymbol{a} \leq -\sqrt{1-\frac{\omega^2(1+\delta)^2}{\gamma^2}}\sin\frac{\omega}{2}+\frac{\omega(1+\delta)}{\gamma}\cos\frac{\omega}{2}, \tag{19}$$

for $\omega \leq \gamma/(1+\delta)$, and

$$\left[\boldsymbol{s}(\omega)\sqrt{1-\frac{\omega^2(1-\delta)^2}{\gamma^2}}-\boldsymbol{c}(\omega)\frac{\omega(1-\delta)}{\gamma}\right]\boldsymbol{a} \geq -\sqrt{1-\frac{\omega^2(1-\delta)^2}{\gamma^2}}\sin\frac{\omega}{2}+\frac{\omega(1-\delta)}{\gamma}\cos\frac{\omega}{2}, \tag{20}$$

for $\omega \in (0, \pi)$. Therefore, the coefficients of the allpass filter $A_N(z)$ that characterize the IIR fullband differentiator with transfer function given by (1) should satisfy linear inequalities given by (17), (18), (19) and (20), and a linear programming techniques [15] can be employed to determine whether such solution exist or not. If not, δ and/or γ needs to be altered.
To obtain the minimum value of δ for known γ such that previous subproblem has a solution, following optimization problem needs to be solved

$$\begin{gathered}\underset{\delta,\,\boldsymbol{a}}{\text{minimize}}\ \delta\\ \text{subject to: } (17), (18), (19), (20)\end{gathered} \tag{21}$$

Since inequalities given by Eqs. (17), (18), (19) and (20) are not linear in δ, some of the one-dimensional search routines [16] can be used to determine the lowest $\delta = \delta_C$ such that previous subproblem has a solution for some γ.

Finally, to obtain the general optimal solution, the value of the parameter γ needs to be determined such that solution to (21) is minimal for the given allpass filter order N,

$$\begin{gathered}\underset{\gamma,\,\boldsymbol{a}}{\text{minimize}}\ \delta\\ \text{subject to: } (21)\end{gathered} \tag{22}$$

and as in the previous subproblem some of the one-dimensional search routines [16] can be utilized.

DESIGN EXAMPLES

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In this section, design examples of the allpass-based IIR fullband differentiators obtained using the proposed design method are discussed for $N=1$ and $N=2$. Note that the order of proposed differentiators equals $2N-1$, while required number of multiplications is $N+1$. As compared to differentiators proposed in [10], proposed differentiators exhibit lower magnitude response error due to existence of one additional design parameter γ. As already mentioned, average group delays of proposed differentiators do not depend on the coefficients values but only on the filter order, note (6). Using (6), phase response linearity error function in degrees can be determined as

$$\xi(\omega) = \frac{180}{\pi}\left[\theta(\omega) - \left(\frac{\pi}{2} - \omega\tau\right)\right], \tag{23}$$

as well as the maximum phase linearity error

$$\eta = \max|\xi(\omega)| \tag{24}$$

Utilization of the proposed design method gives the following transfer function of the first-order fullband differentiators

$$H_1(z) = 1.483614\left(1 - \frac{0.222152+z^{-1}}{1+0.222152z^{-1}}\right). \tag{25}$$

The maximum relative magnitude response and phase linearity errors of the proposed first-order differentiator are equal to $\delta = 5.55\%$ and $\eta = 12.84$ deg. Obtained differentiator has lower maximum relative magnitude response error compared to the third-order differentiator from [2] (5.6%), but slightly worse phase linearity (12.05 deg). On the other hand, proposed differentiator outperforms existing first-order allpass-based solution [12] that has $\delta = 9.03\%$ and $\eta = 15.46$ deg. Magnitude response, $\varepsilon(\omega)$ and $\xi(\omega)$ of considered differentiator are presented in Figure 1.

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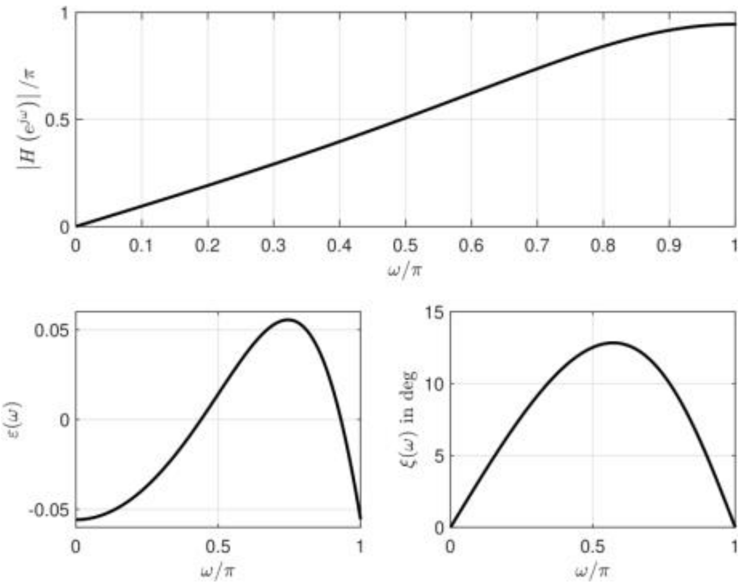

Figure 1. Magnitude response, relative magnitude response error and phase response linearity error of proposed first-order IIR fullband differentiator

Transfer function of proposed third-order IIR fullband differentiator is

$$H_2(z) = 1.524331\left(z^{-1} - \frac{-0.038552+0.278184z^{-1}+z^{-2}}{1+0.278184z^{-1}-0.038582z^{-2}}\right), \quad (26)$$

and its magnitude response, relative magnitude and phase response linearity errors are depicted in Figure 2. Maximum of $\varepsilon(\omega)$ and $\xi(\omega)$ of the proposed third-order differentiator equal $\delta = 2.96\%$ and $\eta = 17.31$ deg.

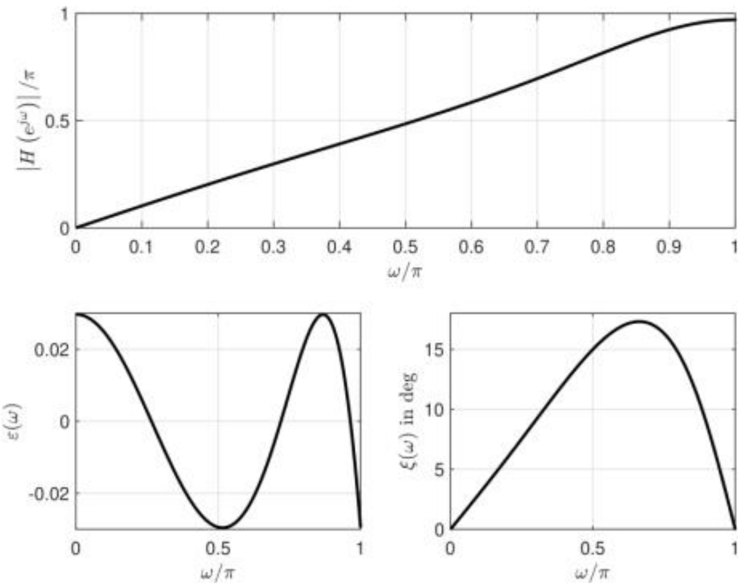

Figure 2. Magnitude response, relative magnitude response error and phase response linearity error of proposed third-order IIR fullband differentiator

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CONCLUSION

In this paper, design of allpass-based IIR fullband differentiators using linear programming is considered. As compared to previous allpass-based solutions, proposed differentiators have an additional design parameter that can be used to further decrease the maximum of the relative magnitude response error. Proposed differentiators have low group delays and obtained first-order differentiator compares favourably with existing higher-order solutions. Higher order differentiators tend to have lower magnitude response errors, but they exhibit higher phase response linearity errors. The future research can extend the proposed approach to design of IIR lowpass and midband differentiators. Given that 5G aims for high efficiency, speed, and accuracy in signal processing, these components can provide significant advantages in achieving these goals.

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